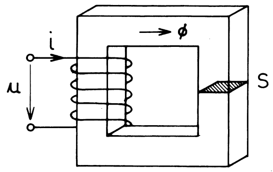
Review of common equations applied in electromagnetic

For uninterrupted magnetic cores the following general equation applies:

$$U_i = -N \cdot \frac{d\Phi}{dt} = -N \cdot S \cdot \frac{dB}{dt} = -N \cdot S \cdot m_o m_r \cdot \frac{dH}{dt}$$

since $H = \frac{I \cdot N}{l_s}$ we may write as follows

$$U_i = -m_o m_r \cdot N^2 \cdot \frac{S}{l} \cdot \frac{di}{dt}$$



In case of sine (harmonic) changes:

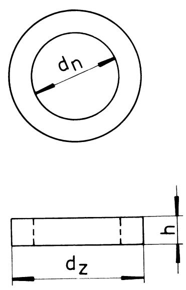
$$U = 4.44 \cdot f \cdot N \cdot S \cdot B = 4.44 \cdot f \cdot m_o m_r \cdot N^2 \cdot \frac{S}{l} \cdot I = 4.44 \cdot f \cdot L \cdot I$$

The product $L = m_o m_r \cdot N^2 \cdot \frac{S}{l}$ represents coil inductance with core.

Due to non-linearity of the hysteresis loop the relative permeability μ_i is not constant but varies with greater voltage or current, therefore neither is inductance a constant quantity. This should be considered when measuring inductance (the above equations do not consider loss in Cu and core). For toroidal cores of rectangular cross-section the following applies:

$$L = 2 \cdot N^2 \cdot h \cdot m_i \cdot 10^{-7} \cdot \ln \frac{d_z}{d_n}$$

where it is usually calculated with initial permeability μ_i.



If the magnetic core with permeability μ_i has an air gap with length **s**, then permeability decreases to μ_i in the equation:

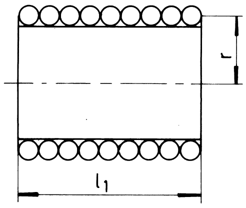
$$m_e = \frac{m_i}{1 + \frac{s}{l} \cdot m_i}$$

the equation is approximate and applies to condition where s<<1. At decrease of permeability there is also a decrease in temperature dependence of the coil, in losses etc.
Inductance of winding with core may be calculated by A_L factor in the equation

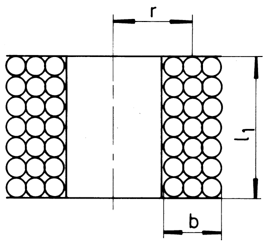
$$L = A_L \cdot N^2$$

A_L factor is dependent on core shape and material permeability and is given by companies for most of ferrite core shapes. For certain characteristic shapes of a coreless coil, inductance may be calculated by the following empirical equation:

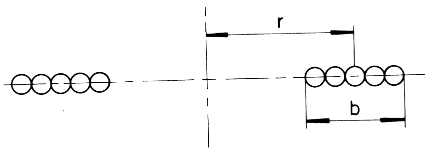
a) single-layer coil $$L_0 = \frac{4.37 \cdot 10^{-3} \cdot (r \cdot N)^2}{r + 1.1 l_1}$$ L_0 [H]; r, l_1 [mm]



b) multi-layer coil $$L_0 = 5.25 \cdot 10^{-3} \cdot \frac{(r \cdot N)^2}{r + 1.5 l_1 + 1.67 b}$$ L_0 [µH]; r, l_1, b [mm]



c) single-layer spiral coil $$L_0 = 3.58 \cdot 10^{-3} \cdot \frac{(r \cdot N)^2}{b + 0.727 \cdot r}$$ L_0 [µH]; r, b [mm]



Magnetic energy of coils is: $$W_L = \frac{I^2 \cdot L}{2} = \frac{B \cdot H}{2} \cdot V$$

Data Sheet

If a rod-shaped core with apparent permeability $[_{app}$ is incorporated in a coreless coil with inductance L_0 we obtain an inductance of coil with core L.

$$L = m_{app} \cdot L_0$$

Value for m_{app} is given by companies as a function of $\frac{l}{d}$ for various material qualities (chart 1)

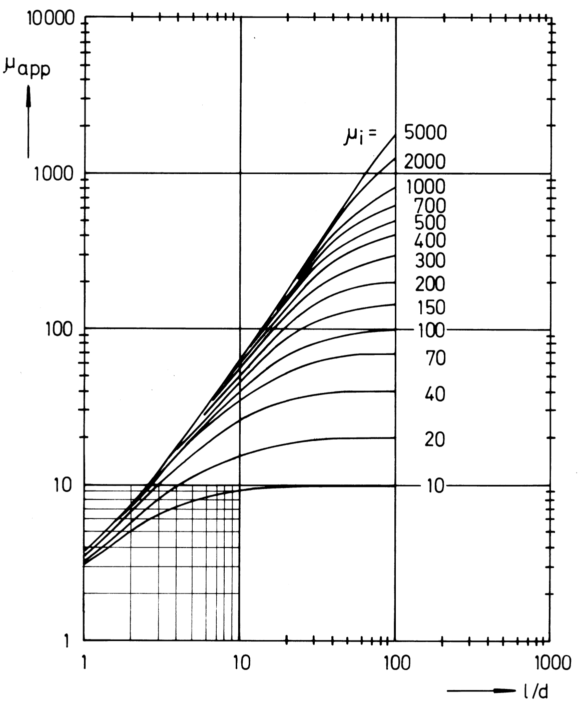


Chart 1

In real coils, losses occur which are shown by heating the coil and the core. Losses occur due to eddy currents in the winding and the core, hysteresis loss of core and to a lesser extent due to so-called residual losses.

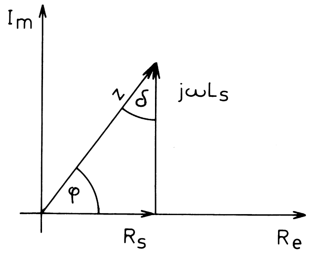
a) Substitute connection of coil with core may be shown with a series substitute connection, impedance of connection being:

$$Z = R_s + jwL_s = R_s \cdot (1 + j\frac{wL_s}{R_s}) = R_s \cdot (1 + jQ) \qquad w = 2p \cdot f$$

therefore absolute impedance value is:

$$|Z| = \sqrt{R_s^2 + w^2 \cdot L_s^2} = R_s \cdot \sqrt{1 + Q^2} = R_s \cdot \sqrt{1 + \frac{l}{D_i^2}} \qquad D_i = \frac{1}{Q} = \frac{R_s}{wL_s}$$

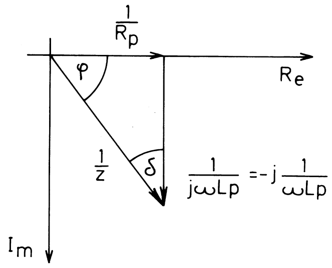
factor $Q = \frac{wL_s}{R_s}$ is called quality factor of coil , D is loss factor $D_i = \frac{1}{Q} = \frac{R_s}{wL_s}$



Parallel substitute connection:

$$Z = wL_p \cdot \frac{Q}{1+Q^2} \cdot (1 + jQ) = wL_p \cdot \frac{D_i}{1+D_i}(1 + j\frac{1}{D_i})$$

absolute value will be: $|Z| = \frac{wL_p}{\sqrt{1+D_i^2}} = \frac{wL_p}{\sqrt{1+w^2 \cdot L_p^2 \cdot G_p^2}}$ where $Q = \frac{1}{D_i} = \frac{R_p}{wL_p}$



From the preceding equations we obtain the following conversions:

$$L_s = \frac{1}{1+D_i^2} \cdot L_p, \qquad R_s = \frac{D_i^2}{1+D_i^2} \cdot R_p, \qquad D_i = \frac{wL_p}{R_p}$$

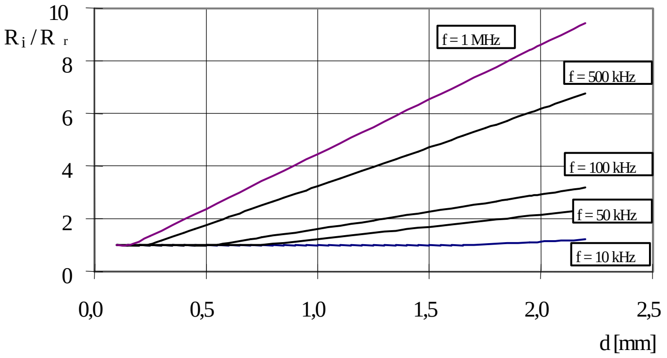
and $L_p = (1+D_i^2) \cdot L_s, \qquad R_p = \frac{1+D_i^2}{D_i^2} \cdot R_s, \qquad D_i = \frac{R_s}{wL_s}$

At fairly small loss factor D_i (at fairly high Q - $Q > 10$) it holds that $L_s = L_p$ and $R_s = R_p$. For coils with higher factor D (at lower Q - $Q < 10$) it holds that equivalent series and parallel components of impedance have no similarity. It is therefore important to give the equivalent substitute connection as well as the measuring frequency in the technical data.

Due to eddy currents the resistance of conductors at a.c. current R_{ac} is greater than for d.c. R_o.

Resistance relation $\frac{R_{ac}}{R_o}$ is dependent on parameter m

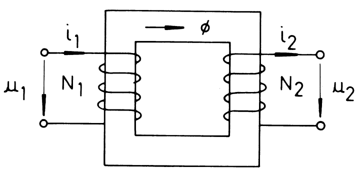
$$m = 0.14 \cdot d \cdot \sqrt{\frac{m \cdot f}{r}}$$

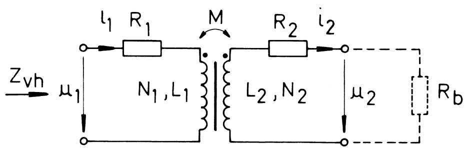


d [cm] - diameter of conductor
f [Hz] - frequency
r [Ωmm^2 /m] - specific resistance
m - permeability

Transformer

Ideal transformer





When transformer loss is not considered and the total magnetic flux Φ flows intact (permeability $m \rightarrow \infty$) through both windings, the transformer is ideal. In this case it holds that:

$$\frac{U_1}{U_2} = \frac{N_1}{N_2} = n, \qquad \frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{1}{n} \text{ on respectively } I_1 \cdot N_1 = I_2 \cdot N_2$$

From the above the following also applies:

$$Z_i = \frac{U_1}{I_1} = n^2 \cdot \frac{U_2}{I_1} = n^2 \cdot R_b$$

If it is considered that permeability of the core has a final value and if loss in windings is also considered, then the following equations apply:

$$M = k \cdot \sqrt{L_1 \cdot L_2}, \qquad s = 1 - k^2 \text{ for } k \approx 1 \text{ it holds that } s = 2 \cdot (1 - k)$$

voltage ratio $\frac{U_2}{U_1} = \frac{N_2}{N_1} \cdot (1-g)$ where factor γ means $g = -2 \cdot \frac{R_2}{R_b} - \frac{1}{2} \cdot (\frac{s \cdot wL_2}{R_b})^2$

Meaning of symbols in the above equations:
k - coupling factor between coils L_1 and L_2
M - mutual inductance
σ - leakage coefficient

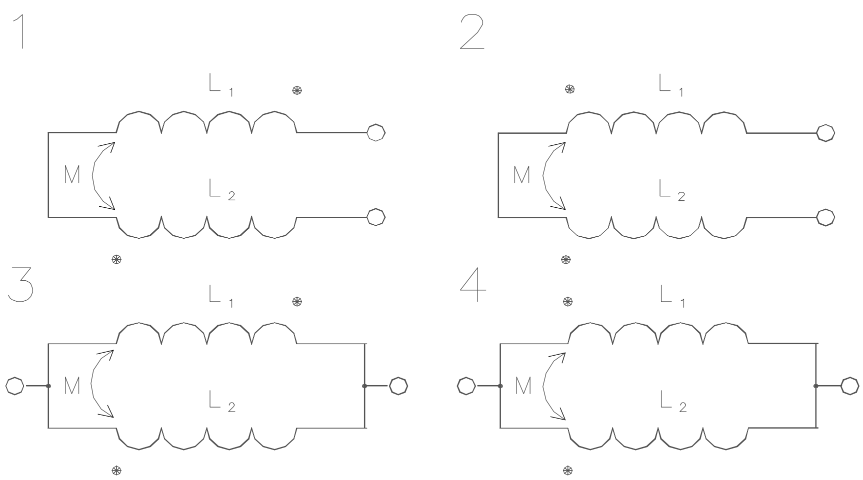
In case of open secondary terminals ($R_b = \infty$) it is: $\frac{U_2}{U_1} = \frac{N_2}{N_1} = \frac{1}{n}$ current ratio $\frac{I_2}{I_1} = \frac{N_1}{N_2} \cdot \frac{k}{1 + \frac{R_b + R_2}{jwL_2}}$

at short-circuited secondary terminals $(R_b = 0)$ we may write:

$$I_1 \cdot N_1 = I_2 \cdot N_2 + I_m \cdot N_1$$

We may note that at open secondary terminals $(R_b = \infty)$, some current I_m, the so-called magnetizing current (since $m_r \neq \infty$) still flows the primary side.

Equivalent inductance of magnetic coil drives



1. Series connection - Circuit 1, Circuit 2
Circuit 1: Fluxes are superposed to each other: $L = L_1 + L_2 + 2M$
Circuit 2: Fluxes are opposite-flowing: $L = L_1 + L_2 - 2M$

2. Shunt connection - Circuit 3, Circuit 4

Circuit 3: Fluxes are superposed to each other: $L = \frac{L_1 \cdot L_2 - M^2}{L_1 \cdot L_2 - 2 \cdot M}$

Circuit 4: Fluxes are opposite-flowing: $L = \frac{L_1 \cdot L_2 - M^2}{L_1 \cdot L_2 + 2 \cdot M}$

In the above equations symbols have the following meaning:
L1, L2, L3 and L4 are inductance’s of single coils
M mutual inductance between both coils

Important Physical constants:

Important physical constants

Gravitational constant	$k = 6.67\text{x}10^{-11}$ [$kg^{-1}m^{3}s^{-2}$]
General gas constant	$R = 8316.96\gamma$ [$kmol^{-1}st^{-1}$]
Avogadro's number	$N_A = 6.0254\text{x}10^{26}$ [$kmol^{-1}$]
Bolzmann constant	$K = 1.38\text{x}10^{-23}\gamma$ [st^{-1}]
Stefan's constant	$\sigma = 5.6696\text{x}10^{-8}$ [Wm^{-2} st^{-4}]
Faraday constant	$F = 9.6519\text{x}10^{7}$ [As $kmol^{-1}$]
Planck's constant	$h = 6.625\text{x}10^{-34}$ [kgm^{2} s^{-1}]
Electron mass	$m_e = 9.1083\text{x}10^{-31}$ [kg]
Electron charge	$e_0 = 1.6\text{x}10^{-19}$ [As]
Light speed in vacuum	$c_0 = 2.9979\text{x}10^{8}$ [ms^{-1}]
Dielectric constant	$e_0 = 8.854\text{x}10^{-12}$ [As $V^{-1}m^{-1}$]
Induction constant	$m_0 = 1.2566\text{x}10^{-6}$ [Vs $A^{-1}m^{-1}$]

European and English units

Length

1 Å (angström) = 10^{-10} [m]

1 µ (micron) = 10^{-6} [m] = 10^{-3} [mm] = $4\text{x}10^{-5}$ [inches]

1 inch (in) = 1/12 [ft] = 25.4 [mm]

1 foot (ft) = 12 [inches] = 304.8 [mm]

1 yard (yd) = 3 [ft] = 914.4 [mm]

1 English mile = 5280 [ft] = 1609.34 [m]

1 sea mile = 6080 [ft]= 1853.18 [m]